2 Defining Fuzzy Sets

Crisp Sets

In classic set theory, an element either belongs to a set ($x \in A$) or it does not ($x \notin A$); there are no in-between possibilities. Sets from the classical theory will be called _____, because they have crisp, well-defined boundaries.

There are three common ways of defining a crisp set A within some universe X.

- Making a _____ of the elements

$$A = \{x_1, x_2, \ldots, x_n\}.$$

- Giving a _____ which the elements must satisfy

$$A = \{x \in X \mid P(x)\}.$$

- Using the _____ function

$$\chi_A : X \to \{0, 1\}, \quad \chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}.$$

Fuzzy Sets

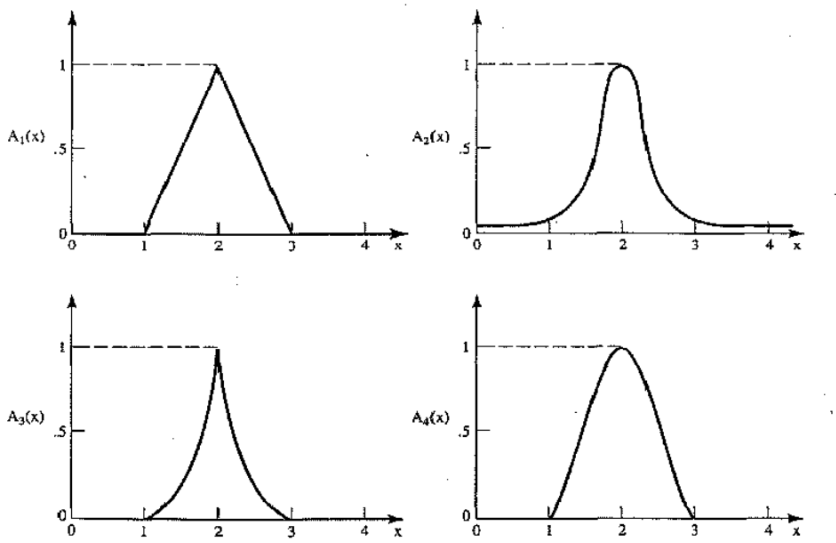
To define a fuzzy set A, we start with a _____ set X for our universe and then decide the _____ of each element x in our set A. Usually, the degrees of membership are numbers between 0 and 1, so that a fuzzy set is a _____

$$A : X \to [0, 1].$$

In our notation, there is no difference between a fuzzy set and its _____ function.

Examples of Fuzzy Sets

The fuzzy set of real numbers close to 2



- Properties of these membership functions:

- The graphs show four possible membership functions – and there are many more possibilities (!)

The fuzzy sets of low, high, and very high education levels.

0 – no education
1 – elementary school
2 – high school
3 – two-year college degree
4 – bachelor's degree
5 – master's degree
6 – doctoral degree

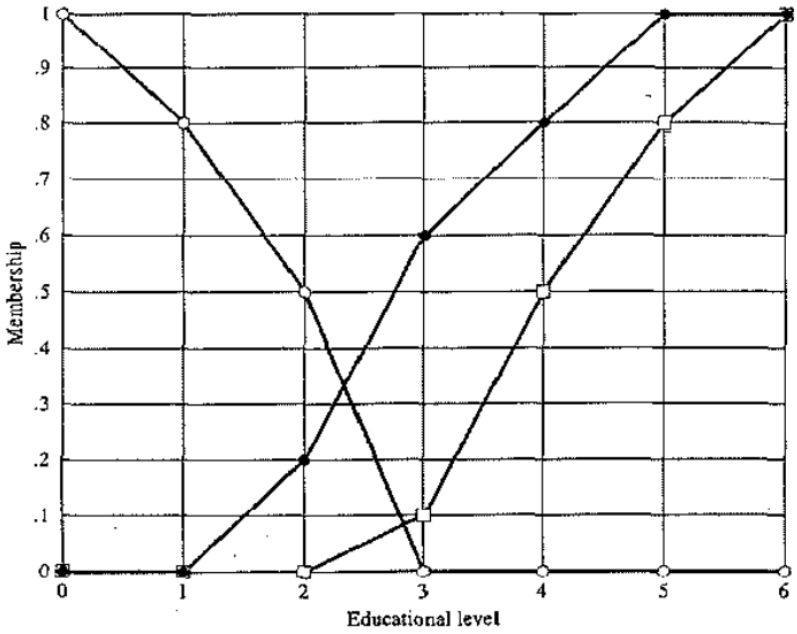


Figure 1.3 Examples of fuzzy sets expressing the concepts of people that are little educated (○), highly educated (●), and very highly educated (□).

- Since these fuzzy sets are finite, we can also define them by the list method

Various levels (medium, high, etc.) for the _____ temperature.

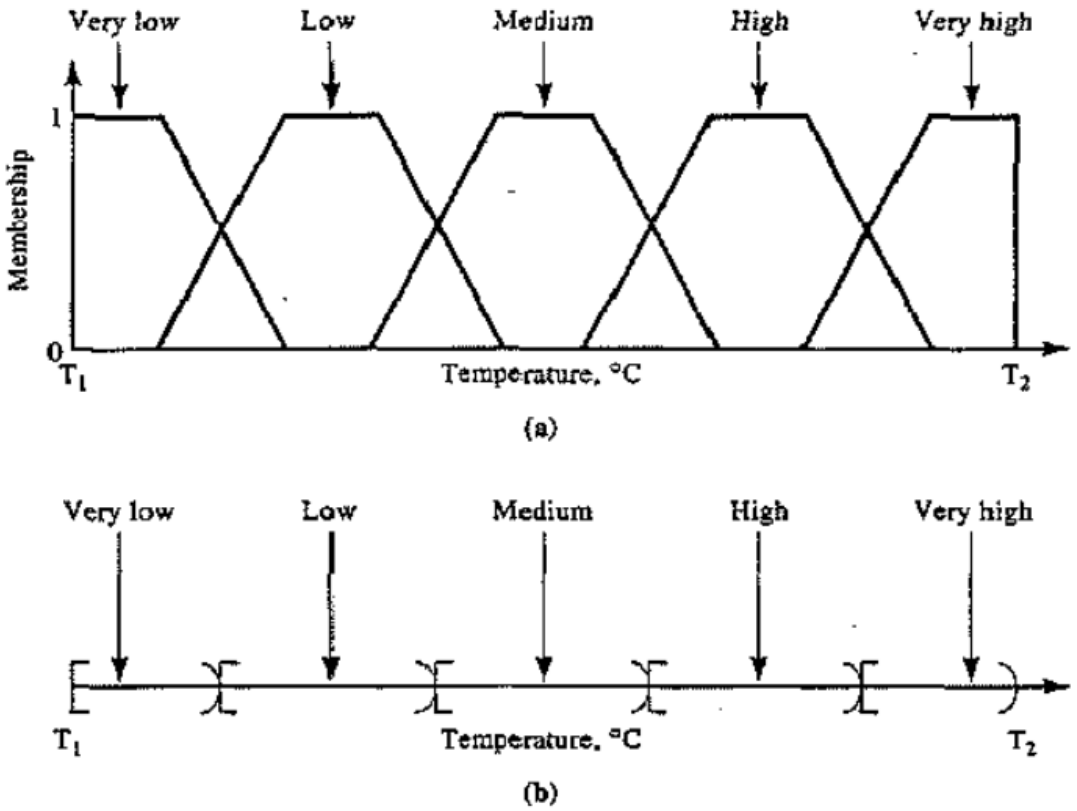


Figure 1.4 Temperature in the range $[T_1, T_2]$ conceived as: (a) a fuzzy variable; (b) a traditional (crisp) variable.

- The definitions are _____-dependent.

- What is the advantage of the fuzzy definition over the crisp definition?

- The most commonly used membership functions in fuzzy set theory are _____ and _____ membership functions.